Residential Electricity Auction with Uniform Pricing and Cost Constraints

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***Abstract*— This paper considers electricity auctions as auctions of a divisible good when the bidders have soft cost constraints. Soft cost constraints are such that the bidder proportionally reduces the desired quantity when the price is over a defined threshold. We prove a policy-consistent mechanism to allocate the goods using uniform pricing. Further, the mechanism is communication efficient and polynomial time computable. We finish by describing how the auction mechanism could work for residential real time electricity pricing using intelligent programmable communicating thermostats.**

I. Introduction

A primary tenet of our economic system is the allocation of goods and services based on cost. There are many cases where an individual may want a certain quantity of an item, but if he cannot get all of what he wants because of budget constraints, he will accept as much as he can afford. Consumable natural resources, such as electricity, gas, and water, are a prime example of such goods. In general, the value they possess comes through achievement of some external goal, and that goal is often proportional, meaning that all is best, but some is good. For instance, consider an increase in the price of gasoline. Generally, the price increases because the supply cannot keep pace with the demand. People do not completely stop driving their cars, most just drive less, therefore consuming less gasoline. Anecdotally at least, they buy as much fuel as they can afford (e.g. Bill buys $20 of gasoline regardless of the volume $20 buys). In this vein, we study auctions of an infinitely divisible good to bidders with soft constraints on total cost.

The primary motivation for this work is real time residential electricity pricing. In [1], Severin Borenstein advocates for consumer real time electricity pricing that couples the consumer price to the generation and distribution markets. In California there are a number of time varying pricing programs in place, but the pricing usually follows some predetermined schedule with the possibility of a few extreme pricing instances each year. Further, the likelihood of such a program becoming commonplace in the state of California is very high. We want to take it a step further, and our goal, through this work, is to enable automatic residential real time electricity pricing.

II. Problem Description

We have removed the complexity of how, and at what price, the product has been created. We simply assume that the product is divisible and scarce, in that we only have a finite quantity to be allocated in whatever way we choose. Further, we assume that there are many bidders desiring the item. Each bidder desires a certain quantity of the item, and they are willing to pay some maximum cost. If they cannot get their desired amount because the price is too high, then they want to get as much as they can afford.

The goal of this research is to uncover an auction mechanism for allocating the scarce resource to the bidders adhering to their cost constraints. The mechanism should compute the clearing price off-line using only a single bid from each bidder. For reasons of simplicity and perceived fairness we use uniform pricing, so that each bidder pays the same price. Further, we want a policy-consistent algorithm that elicits the bidders true desires.

III. Previous Work

Our treatment of this issue is from a game theoretic perspective, and for an excellent modern discussion of algorithmic game theory see [2]. Game theory formalizes the tools for analyzing how rational agents interact to achieve their goals from an economic point of view. The game theoretic issue of algorithmic mechanism design is concerned with developing the framework for agent interaction such that certain criteria are met.

A search of the literature uncovered no previous research on this exact problem. There is a wealth of literature on auctioning divisible goods, mainly dealing with treasury bonds. Further, a few researchers have worked on budget constraints, but they usually take the form of hard constraints where the item is not desired above a certain price. Electricity auctions are also a hot topic, but mainly focusing on generation scheduling.

The auctioning of multi-unit and divisible goods has been discussed for quite some time. In 1979 Robert Wilson wrote about auctions of divisible goods in [3]. He considered both uniform pricing and discriminatory pricing. Under uniform pricing, all winners pay the same unit price for the item, and under discriminatory pricing, each winner potentially pays a different price, as with a Vickrey-Clark-Groves (VCG) mechanism. He concluded that discriminatory pricing is not better than uniform pricing, and it could in fact be worse. But his is not the final word on this issue.

In 1993 Back and Zender described the possibility of "collusion like equilibria" in divisible goods auctions under uniform pricing, [4]. With these detrimental equilibria, the auction revenue could be arbitrarily low, but by 2001 they had figured out a way around this issue by allowing the auctioneer the option of strategic supply withdrawal, [5]. Further, in 2002, Wang and Zender wrote about treasury auctions in [6]. With treasury auctions, each bidder receives the same value of the item, but the true value is not known a prioi. They also found undesirable revenue with uniform pricing. In 2004, Kremer and Nyborg deal with collusion like equilibria in [7]. Their findings show that these unwanted equilibria exist only under a continuous bidding strategy. If the bidding is restricted to finite steps, the underpricing can be well controlled.

There is limited literature on budget constrained auctions, and as mentioned previously, most research has focused on hard constraints where the bidder does not want the item for greater than a certain cost. In 2000, Eric Maskin investigated budget constrained single good auctions in [8]. An interesting finding was that these auctions lose efficiency under budget constraints. Benoit and Krishna examine budget constrained auctions of multiple goods in a 2001 paper, [9]. They consider a model where there are many bidders and only two non-divisible items. They noticed that cross bidding can effect the price paid for each item. In 2005, Christian Borgs et al. considered the problem of multiple identical items for auction to a multitude of budget constrained bidders, [10]. The bidders either want a subset of items below their budget, or they want no items. The classic VCG mechanism is not incentive compatible in this case because the utilities are not quasi-linear. They found that there is no deterministic truthful mechanism that can allocate all of the units, but they demonstrated a method to allocate some of the units truthfully with maximized revenue. Dobzinski, Lavi, and Nisan recently (2008) published an impossibility proof for multi-unit constrained budget auctions, [11]. They consider a budget in which each bidder has a value for the item and a maximum payment they are willing to make. They will take some fraction of the items at any price below the budget, but they do not want any if the price exceeds the budget. They find that it is impossible to get pareto-optimality and truthfulness without public budgets.

Electricity market deregulations of the late 1990's and the California electricity crisis of 2000 and 2001 sparked a bit of research regarding electricity auctions. Chao and Peck's 1996 paper, [12], discusses the design of electricity transmission markets. Hobbs et al. describe a VCG mechanism for electricity supply and demand markets in a 2000 paper, [13]. In a 2002 paper by Fabra et al. modeling of electricity markets is discussed in detail, [14]. They discuss uniform pricing and discriminatory pricing in the forms of VCG and pay-as-bid mechanism, among other things. They also describe how the "collusion like equilibria" can be thwarted with discrete bidding functions in the context of electricity markets. They seem to prefer VCG auctions to the alternatives. The debate over uniform pricing versus discriminatory pricing continues with the 2003 paper by Zang and colleagues, [15], and they found no clear cut advantages either way. Finally, Severin Borenstein has a wonderful treatise on the problems with electricity markets where he advocates for, among other things, consumer real-time pricing, [1].

IV. Policy-Consistent Clearing Algorithm

The punch line: We have formulated a policy-consistent auction mechanism for allocating all of the goods under soft budget constraints. The clearing price is computed in polynomial time, and it only requires a single bid from each bidder. The mechanism was inspired by the Shapley Value cost sharing mechanism described in [16] and the real-time dynamic wireless spectrum auctions in [17], but it bears little actual resemblance to either.

To define the problem more succinctly, consider that there is a divisible item E^* with many bidders ($n > 1$). Each bidder ($i = 1, 2, ..., n$) has a private evaluation of the maximum desired quantity of the item, α_i, and a maximum unit price for that quantity, ρ_i. The evaluations are such that the total desires of the bidders cannot be met with the supply, that is, $\sum_{i=1}^{n} \alpha_i > E^*$. Furthermore, the bidder exhibits a *soft budget constraint* (see Definition IV.1) in that if the price (P) is too high ($P > \rho_i$), then the bidder wants the largest amount she can get given her budget constraint $\alpha_i \rho_i$. The goal is to allocate the full quantitiy to the bidders, $A = a_1, a_2, ..., a_n$, at a uniform price P.

Definition IV.1 *Soft Budget Constraint*

A budget in which the bidder desires a given quantity α_i of an item, for a maximum price ρ_i, but will accept a proportionally lower quantity at a higher price exhibits a soft budget constraint. The actual quantitiy desired is represented by a_i, and the actual price is given by P. A soft budget constraint function, $\beta_i : P \rightarrow a_i$, is succinctly represented by Equation 1.

$$\beta(P) = \begin{cases} \frac{\rho_i \alpha_i}{P} & ; P \geq \rho_i \\ \alpha_i & ; P < \rho_i \end{cases} \tag{1}$$

A. Soft Budget Constrained Mechanism

The soft budget constrained mechanism accepts a single bid, consisting of the maximum quantity and the maximum price, from each bidder. It returns the clearing price and allocations to the bidders. Proposition IV.2 describes the mechanism in full.

Proposition IV.2 *Soft Budget Constrained Mechanism*

The soft budget constrained mechanism, f, accepts bids and returns a clearing price and allocation for the item: $f : \{E^*, b_1, b_2, ..., b_n\} \rightarrow \{P, a_1, a_2, ..., a_n\}$. The bids consist of the maximum quantity and maximum price for that quantity, $b_i = \{\alpha_i, \rho_i\}$. There are three steps to the soft budget constrained mechanism:

1) *Order the bids based on the maximum price in ascending order placing a fictitious bid, $b_{n+1} = \{0, \infty\}$, at the end, Equation 2.*

$$\rho_1 \leq \rho_2 \leq ... \leq \rho_n \leq \rho_{n+1} \tag{2}$$

2) *Iterate on Equations 3, starting from $k = 1$ until $\rho_{k-1} < P \leq \rho_k$. (Note that there could be numerical concerns because the denominator starts out negative and through successive iterations becomes positive. A divide by zero could occur. This is easily enough remedied by checking feasibility before dividing.) The fictitious bid ensures that the algorithm stops.*

$$\begin{aligned} k \leftarrow & \quad k+1 \\ P \leftarrow & \quad \frac{\sum_{i=1}^{k-1} \rho_i \alpha_i}{E^* - \sum_{i=k}^{n} \alpha_i} \end{aligned} \tag{3}$$

3) *Compute the allocations using Equations 4 and 5.*

$$a_i = \frac{\rho_i \alpha_i}{P} \quad \text{for } i = 1, 2, ..., k-1 \tag{4}$$

$$a_i = \alpha_i \quad \text{for } i = k, k+1, ..., n \tag{5}$$

The first step of the mechanism orders the bids by the maximum bid price. Next, it constrains the allocation to the lower priced bidders ($i < k$, the numerator) so that they get the maximum that their soft budget constraint allows. It leaves the higher priced bidders' allocation at the maximum ($i \geq k$, the denominator), and it recomputes the price. This continues until the price only constrains the $k-1$ bidders. Finally, the allocation is computed for the constrained bidders, and the unconstrained bidders receive their full allocation.

To see why the algorithm works, we need to take a look at the computation for the total allocation under uniform pricing with constrained allocation to some of the bidders, Equation 6. The bidders $i = 1, 2, ..., k-1$ have their allocation reduced by their budget constraint, and the remaining $i = k, k+1, ..., n$ bidders obtain their full desired allocation. A good allocation occurs when the price P only constrains the $i = 1, 2, ..., k-1$ bidders.

$$E^* = \sum_{i=k}^{n} \alpha_i + \frac{1}{P} \sum_{i=1}^{k-1} \alpha_i \rho_i \tag{6}$$

It is obvious that this mechanism can be computed in polynomial time. Further, it only requires a single bid from each bidder, making it communication efficient.

B. Policy Consistency

An algorithm is policy-consistent if it coaxes the bidders to directly reveal their true valuations of the item. This coaxing comes in the form of their not being able to get a more individually favorable outcome by revealing false information. The following formal definition is taken from [18].

Definition IV.3 *Incentive Compatibility*

A mechanism $(f, b_1, b_2, ..., b_n)$ is called incentive compatible if for every player i, every $v_1 \in V_1, v_2 \in V_2, ..., v_n \in V_n$ and every $\tilde{v}_i \in V_i$, if we denote $A = f(v_i, v_i)$ and $\tilde{A} = f(\tilde{v}_i, v_i)$, then $v_i(A) - p_i(v_i, v_i) \geq \tilde{v}_i(\tilde{A}) - p_i(\tilde{v}_i, v_i)$.

The soft budget constrained mechanism is policy-consistent, but before we get to the formal proof we need a few more concepts. Each bidder gains some utility from the allocation, and the utility takes the normal form of valuation, v_i, minus cost, p_i, as in Equations 7, 8, and 9.

$$u_i = v_i - p_i \tag{7}$$

$$v_i(a_i, P) = \begin{cases} \min\{a_i \rho_i, \alpha_i \rho_i\} & ; P \leq \rho_i \\ \min\{a_i P + a_i - \frac{\alpha_i \rho_i}{P}, \alpha_i \rho_i\} & ; P > \rho_i \end{cases} \tag{8}$$

$$p_i = P a_i \tag{9}$$

Because of the inherent nonlinearities, finding an appropriate valuation for the soft budget constraint was not straightforward. In general, this valuation ensures that the utility is zero along the budget constraint. To see how it works, let's look at a few cases of the total utility under varying circumstances:

- When the the price is below the maximum per-unit price with full or partial allocation, the utility evaluates to 0.
- When the price is below the maximum per-unit price with greater than full allocation, the utility is given by $u_i = \alpha_i \rho_i - P a_i$. This becomes negative if the total cost exceeds the budget constraint and is positive if the bidder get more for less. A bidder does not need any more than her maximum, but she will accept more if the cost is low enough.
- If the price is higher than the maximum per-unit price, and the allocation is along the budget constraint given in Equation 4, then the utility is zero.
- If the per-unit price is higher, and the allocation is below the budget constrained allocation, then the utility is negative because a bidder wants as much as she can get.
- If the price is higher, and the allocation is greater than the budget constrained allocation, the utility is negative because the total cost exceeds the budget.

With the goal of maximizing the utility of each agent, we are now ready for the theorem.

Theorem IV.4 *Policy Consistency of Soft Budget Constrained Mechanism*

The Soft Budget Constraint Mechanism as outlined in Proposition IV.2 is policy-consistent when the bidders have soft budget constraints as defined in Definition IV.1.

Proof:

The proof consists of a series of inequalities evaluated for the cases of constrained allocation and unconstrained allocation. Throughout the proof, we will consider the case when the bid consists of truthful revelation with a hat and any other bid with a tilde.

- Truthful revelation variables: $\hat{\rho}_i, \hat{\alpha}_i, \hat{P}, \hat{a}_i, \hat{u}_i$

- Untruthful revelation variables: $\tilde{\rho}_i, \tilde{\alpha}_i, \tilde{P}, \tilde{a}_i, \tilde{u}_i$

Case 1 ($i \geq k$): We begin with the unconstrained allocation where each unit receives the maximum allocation at a price below their maximum price. The truthfully revealed allocation is $\hat{a}_i = \hat{\alpha}_i$, and the utility is given by Equation 10.

$$\begin{aligned} \hat{u}_i &= \min\{\hat{\alpha}_i\hat{P}, \hat{\alpha}_i\hat{\rho}_i\} - \hat{P}\hat{\alpha}_i \\ \hat{P} &\leq \hat{\rho}_i \\ \implies \hat{u}_i &= 0 \end{aligned} \tag{10}$$

For the false bid, the utility is given as follows:

$$\tilde{u}_i = \min\{\tilde{\alpha}_i\tilde{P}, \hat{\alpha}_i\hat{\rho}_i\} - \tilde{P}\tilde{\alpha}_i$$

For $\tilde{\alpha}_i > \hat{\alpha}_i \implies \tilde{P} > \hat{P}$

$$\begin{aligned} \tilde{\alpha}_i\tilde{P} &\geq \hat{\alpha}_i\hat{P} \\ \implies \tilde{u}_i &= (0 \text{ or } \hat{\alpha}_i\hat{\rho}_i - \tilde{P}\tilde{\rho}_i) \leq 0 \end{aligned} \tag{11}$$

For $\tilde{\alpha}_i < \hat{\alpha}_i \implies \tilde{P} < \hat{P} < \hat{\rho}_i$

$$\begin{aligned} \implies \tilde{\alpha}_i\tilde{P} &\leq \hat{\alpha}_i\hat{\rho}_i \\ \implies \tilde{u}_i &= \tilde{\alpha}_i\tilde{P} - \tilde{\alpha}_i\tilde{P} = 0 \end{aligned} \tag{12}$$

Equations 10, 11, and 12 $\implies \tilde{u}_i \leq \hat{u}_i \implies$ no better off by lying!

Case 2 ($i < k$): We continue the proof for the case when the allocation is constrained. In this case, the price is larger than the maximum bid price, and the allocation is constrained.
The truthfully revealed allocation is constrained with the utility given by Equation 13.

$$\begin{aligned} \hat{u}_i &= \min\{\hat{a}_i\hat{P} + \hat{a}_i - \frac{\hat{\alpha}_i\hat{\rho}_i}{\hat{P}}, \hat{\alpha}_i\hat{\rho}_i\} - \hat{P}\hat{a}_i \\ \hat{a}_i &= \frac{\hat{\alpha}_i\hat{\rho}_i}{\hat{P}} \\ \implies \hat{u}_i &= 0 \end{aligned} \tag{13}$$

For the false bid, the allocation and utility are given as follows:

$$\begin{aligned} \tilde{a}_i &= \frac{\tilde{\alpha}_i\tilde{\rho}_i}{\tilde{P}} \\ \tilde{u}_i &= \min\{\tilde{\alpha}_i\tilde{\rho} + \frac{\tilde{\alpha}_i\tilde{\rho}_i}{\tilde{P}} - \frac{\hat{\alpha}_i\hat{\rho}_i}{\tilde{P}}, \hat{\alpha}_i\hat{\rho}_i\} - \tilde{P}\tilde{a}_i \end{aligned} \tag{14}$$

For $\tilde{\alpha}_i\tilde{\rho}_i \geq \hat{\alpha}_i\hat{\rho}_i \implies \tilde{P} \geq \hat{P}$.

$$\begin{aligned} \tilde{\alpha}_i\tilde{\rho}_i + \tfrac{\tilde{\alpha}_i\tilde{\rho}_i}{\tilde{P}} - \tfrac{\hat{\alpha}_i\hat{\rho}_i}{\tilde{P}} &\geq \hat{\alpha}_i\hat{\rho}_i \\ \implies \tilde{u}_i = \hat{\alpha}_i\hat{\rho}_i - \tilde{\alpha}_i\tilde{\rho}_i &\leq 0 \end{aligned} \tag{15}$$

For $\tilde{\alpha}_i\tilde{\rho}_i < \hat{\alpha}_i\hat{\rho}_i \implies \tilde{P} < \hat{P}$.

$$\begin{aligned} \tilde{\alpha}_i\tilde{\rho}_i + \tfrac{\tilde{\alpha}_i\tilde{\rho}_i}{\tilde{P}} - \tfrac{\hat{\alpha}_i\hat{\rho}_i}{\tilde{P}} &< \hat{\alpha}_i\hat{\rho}_i \\ \tfrac{\tilde{\alpha}_i\tilde{\rho}_i}{\tilde{P}} - \tfrac{\hat{\alpha}_i\hat{\rho}_i}{\tilde{P}} &< 0 \\ \implies \tilde{u}_i < \hat{\alpha}_i\hat{\rho}_i - \tilde{\alpha}_i\tilde{\rho}_i &= 0 \end{aligned} \tag{16}$$

Finally Equations 13, 15, and 16 $\implies \tilde{u}_i \leq \hat{u}_i \implies$ again the bidder is no better off by lying!

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V. APPLICATION

The soft budget constrained mechanism was envisioned with residential real time electricity markets in mind. The key idea is to treat electricity consumption over a fixed time period as a scarce resource and auction the resource to individual energy consumers. The auction would operate as follows:

1) Bid Call: At a predefined time before the start of the auction period, the units (e.g., homes, refrigerators, thermostats, etc.) in the residential market must submit bids. The bids consist of the maximum expected energy consumption and the maximum price the unit is willing to pay.
2) Clearing: The soft budget constrained mechanism computes the clearing price and returns the price and allocation to the units before the start of the auction period.
3) Auction Period: At the start of the auction period, the price goes into effect and lasts the predefined length of time. The units are charged the clearing price for their electricity consumption.

For a concrete example of how residential real time electricity auctions could work using this scheme, consider a market for intelligent programmable communicating thermostats. The market could operate with real time prices determined using the soft budget constrained mechanism on 15 minute intervals. The bids would be due 5 minutes before the start of each auction period. The quantity of constrained resource, that is, MWh electricity, for each 15 minute period would be determined based on supply and demand in the generation and distribution markets. The thermostats need two things to compete in this auction setting: price responsiveness and energy consumption prediction.

In [19] we outlined a method of controlling heating ventilation and air conditioning (HVAC) compressors using low frequency pulse width modulation (PWM). Low frequency PWM turns the on/off control signal to an HVAC compressor into a proportional signal at a low sample rate, typically 15min. Temperature control can be accomplished using any type of controller that outputs a proportional signal.

Low frequency PWM enables thermostat price responsiveness. The power consumption of HVAC compressors is approximately proportional to the compressor duty cycle. If the PWM of the HVAC system was synchronized with the auction period, the energy consumption could be controlled using tunable saturation of the control signal, giving the unit price responsiveness.

Energy prediction is also enhanced by low frequency PWM. Low frequency PWM has a linearizing effect on HVAC systems that makes on-line system identification simpler than with traditional control strategies. With an accurately identified system, the expected energy consumption could be bid based on one step look ahead using the model. In an even more simple scenario, the unit could operate with a time delay equal to the time between the bid call and the start of the auction period (5min). That is, the control action would be computed before the bid due time, submitted for maximum power in the bid, and not performed until the start of the auction period. Either method would result in accurate power prediction.

The soft budget constrained mechanism is perfectly suited to residential real time electricity markets for a number of reasons. The bidding language is simple enough to require very little communication expense yet expressive enough to define the demand over a continuous range of prices. Further, the bidding language relates directly to human notions of cost and, therefore, is easy for users to understand. The mechanism is fast enough and communication efficient enough that communication and computation take a trivial amount of time. Finally, the speed allows the bids to be accepted very close to the start of the auction period, which improves the bidding unit's energy prediction.

VI. Discussion

Previous research may seem to suggest a policy-consistent divisible item auction with uniform pricing and bidder cost constraints is impossible, but we have shown that it, on the contrary, is possible. As noted in [4] and others, collusion like equilibria are a strong possibility with uniform pricing in divisible goods markets. Up until this point, the two primary defenses to these disastrous equilibria have been discrete bidding functions and the option of strategic reduction of the item's quantity.

Our positive result is based on the assumption that the bidders exhibit soft cost constraint. The collusion like equilibria were seen when the bidding strategy was arbitrary and continuous, which allowed the bidders to reveal demand curves that were very steep around equilibrium. The soft cost constraint is explicitly not arbitrary because it fixes the strategy over a continuum based on two variables. Further, it is not steep around the equilibrium point. Therefore, in this special case, collusion like equilibria do not apply.

This treatment of the issue did not directly consider the main criticism of uniform pricing, low revenue. In our formulation there is no explicit way to examine revenue because the true value of the item is not considered, only that it is scarce. Therefore, the value is strictly determined by the bidders' willingness to pay. On the other hand, it is unlikely that low revenue would be a problem unless the bidders strictly undervalued the item, and if that were the case, the item would not be scarce.

Finally, the impossibility proof of multi-unit auctions with constrained budgets described in [11] has a foreboding presence because it says that optimality and truthfulness are impossible. Dobzinski et al., however, considered a very different type of cost constraint than considered here. Their constraint was hard, meaning that above a maximum unit price, the bidder no longer wanted the item. The soft budget constraint considered in our treatment proportionally reduces the desired amount above the price threshold. The difference in the constraints most likely explains our conflicting results.

VII. Conclusion

The overreaching goal of this research was to enable automatic real-time consumer electricity pricing. With that goal in mind we studied scarce resource auctions, that is auctions of a divisible good when the demand outstrips supply. Further, we introduced a new budget constraint concept – the soft budget constraint – where, if the price is too high, the bidder wants as much of the item as he can afford.

We have described a polynomial time computable mechanism to allocate a scarce divisible good under soft budget constraints. Further, the mechanism only requires a single bid from each of the bidders making it communication efficient. The key positive result is that our mechanism elicits truthful responses from the bidders.

The soft budget constrained mechanism offers much promise for enabling automatic real time consumer electricity pricing. Revenue concerns are the key unanswered question with this scheme, and they are a quite interesting and nuanced topic. Residential real time pricing and electricity auctions should be introduced to reduce generation costs by lowering the peak load, among other things. Further, under real time pricing the revenue must be sufficient to maintain (or improve) profitability. We do not address these issues here, but this line of reasoning points our direction for future research.

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