Low-Frequency Pulse Width Modulation Design for HVAC Compressors

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Introduction

Residential HVAC Compressor Types

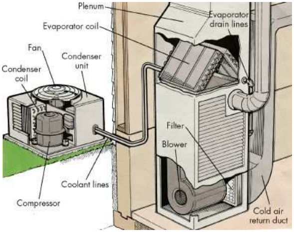
- Single speed - most common
- Multi-speed - runs at multiple powers
- Continuous - not common

Residential HVAC Control

- Hysteresis Control
- Cycling at low frequency (4-6/h)

Proposition

- Cycle using PWM
- Control with Time Invariant Controls
- Simplify operation of multi-stage units
- Simplify control of power consumption.
- Previous work (Federspiel, Lanning, Li, & Auslander, 2001)



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Motivation – Peak Power

We don't generate enough power!

- "Flex your power" Days
- Brown-outs
- Rolling blackouts

Peak Power is Dirty and Expensive!

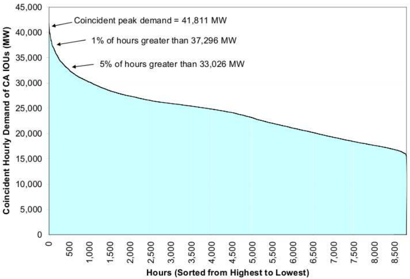
- Peaker Plants
- Pollution
- Carbon Emissions

Global Problem

- China
- South Africa
- United States

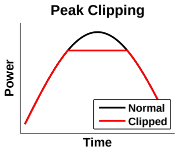
Overall Goal: Reduce the peak power

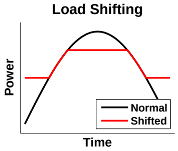
2004 Load Duration for CA IOU

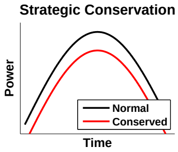


Motivation – Load Management

- LM Goal:
  - Manipulate power demand on the electrical distribution and generation system.
- LM Types (Bellarmine, 2000)
  - Peak Clipping
  - Load Shifting
  - Strategic Conservation
- Reasons to use LM:
  - Avoid blackouts
  - Avoid peaker plants
- Examples Technologies:
  - Load Switches (Navid-Azarbaijani & Banakar, 1996)
  - Grid Friendly Appliances
  - AutoDR







Theoretical Basis – PWM

Pulse Width Modulation (PWM)

- Continuous output from ON/OFF actuation signal
- Fixed period, T, pulse train with varying High-Time
- Specified by duty cycle, $\phi(kT)$
- Direction indicated by second signal
- PWM signal, $u(t)$, is non-linear!

$$\phi(kT) = \begin{cases} \frac{t_{on}}{T} & \text{for positive actuation} \\ -\frac{t_{on}}{T} & \text{for negative actuation} \end{cases}$$

$$u(t) = \begin{cases} U_{max}\, sgn(\phi) & \text{for } kT \leq t < kT + |\phi(kT)|T \\ 0 & \text{for } t \geq kT + |\phi(kT)|T \end{cases}$$

Theoretical Basis – System Response

System Definition

- Consider linear time invariant continuous time system
- Standard state-space representation [A,B,C,D]

$$\begin{array}{lll} \frac{d}{dt}x = Ax(t) + Bu(t) & A \in \Re^{nxn} & B \in \Re^{nxp} \\ y = Cx(t) + Du(t) & C \in \Re^{mxn} & D \in \Re^{mxp} \end{array}$$

System Response to PWM

- Equation for low pulse; free response
- Equation for high pulse; forced response

$$x(t) = \begin{cases} e^{A(t-kT)}x(kT) + \int_{kT}^{t} e^{A(t-\tau)} BU_{max} sgn(\phi(k)) d\tau \\ \qquad \text{for } kT < t \leq kT + |\phi(k)|T \\ e^{A(t-kT-|\phi(k)|T)}x(kT - |\phi(k)|T) \\ \qquad \text{for } t > kT + |\phi(k)|T \end{cases}$$

Theoretical Basis – Linearized Response

Discretize at PWM time (T)

- Single equation
- Non-linear input $h(kT, u)$

$$\begin{aligned} x((k+1)T) &= A_d x(kT) + h(kT, u) \\ A_d &= e^{AT} \\ h(kT, u) &= e^{AT}(I - e^{-AT|\phi(k)|})A^{-1}BU_{max}\, sgn(\phi(k)) \end{aligned}$$

Non-Linear System Equation

Valid Input Linearization

- Traditionally, when T is small
- Actually, AT must be small
- Therefore, A can be small with large T

$$\begin{aligned} x(k+1) &= A_d x(k) + \hat{B}_d \phi(k) \\ A_d &= e^{AT} \\ \hat{B}_d &= (e^{AT} - I)A^{-1}BU_{max} \end{aligned}$$

Linearized System Equation

System Description

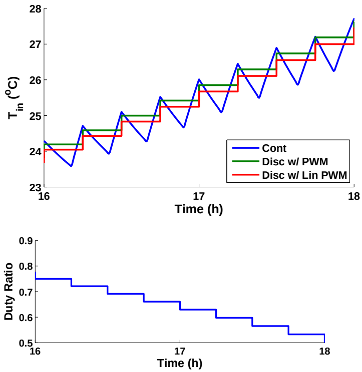
Goal: Reference following

- Controller Design
  - First Order MISO System
  - u_{dr}: compressor duty cycle ($M = 4000$)
  - T_{out}: Outside temperature
  - T_{in}: Inside temperature

$$\begin{aligned} x(k+1) &= A_d x(k) + \hat{B}_d \phi(k) \\ y(k) &= x(k) = T_{in}(k) \\ A_d &= [0.69768] \\ \hat{B}_d &= [0.30232, -7.5581] \\ \phi(k) &= [T_{out}, u_{dr}]^T \end{aligned}$$

- Design Verification
  - House simulation software (Burke & Auslander, 2008)

System Response



Output Filter Design

Filter Needs

- Synchronous with PWM, Low Pass
- Performance Specification:
  - $y_f(kT)$: filtered signal
  - $y(t)$: continuous time signal

$$\| e_f \| = \left(\sum_{k=0}^{\infty} \sum_{i=0}^{T/T_s - 1} \left(y(kT - iT_s) - y_f(kT) \right)^2 \right)^{1/2}$$

Parametric Filter Design

- Digital filters with $T_s = 5s$
- Butterworth (IIR)
  - ω_n: cutoff frequency
  - n: system order
- Boxcar (FIR)
  - n: system order
- Results not surprising given error spec.

Parametric Filter Study

Type	**n**	ω_n	$\| e_f \|$
butter	3	0.0007937	99.721
butter	3	0.0009259	92.55
butter	3	0.0011111	84.306
butter	3	0.0013889	75.853
butter	3	0.0018519	69.395
butter	3	0.0027778	67.069
butter	3	0.0055556	79.135
butter	1	0.0013889	65.648
butter	2	0.0013889	67.298
butter	4	0.0013889	87.528
butter	5	0.0013889	99.994
butter	6	0.0013889	111.29
boxcar	30	-	102.76
boxcar	60	-	84.633
boxcar	90	-	73.561
boxcar	120	-	68.748
boxcar	150	-	65.659
boxcar	180	-	**64.042**

Controller Design

Proportional plus Integral (PI) Controller

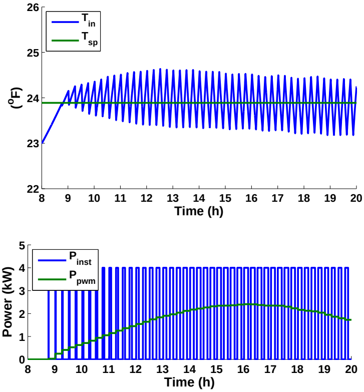
$$\begin{aligned} e(k) &= y_{ref}(k) - y_f(k) \\ e_{int}(k) &= e_{int}(k-1) + e(k)T \\ P(k) &= k_p e(k) + k_i e_{int}(k) \end{aligned} \quad (1)$$

Anti-Windup

$$e_{int}(k) = \begin{cases} \frac{P_{max} - k_p e(k)}{k_i} & P(k) > P_{max} \\ \frac{P_{min} - k_p e(k)}{k_i} & P(k) < P_{min} \end{cases} \quad (2)$$

Iterative controller design

PWM Control of First Order System



Multi Stage Units

Multi-Stage Systems

- Multiple output powers
- Significant efficiency advantages
- Traditionally hard to control

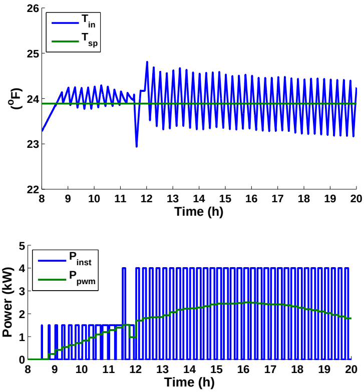
Control with Low-Freq PWM

- Use largest stage with power less than $P(k+1)$

$$P_{cur}(k+1) = \begin{cases} P_1 & 0 < P(k+1) \leq P_1 \\ P_2 & P_1 < P(k+1) \leq P_2 \end{cases}$$

$$u_{dr}(k+1) = P(k+1)/P_{cur}(k+1)$$

PWM Control with 2 Stage Compressor



Tunable Saturation

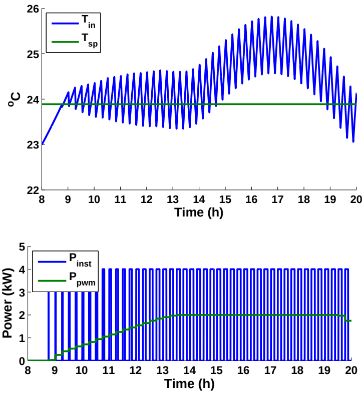
Direct Load Control (DLC)

- Radio operated switch (Navid-Azarbaijani & Banakar, 1996)
- Cuts power from compressor for specified time
- Variable effect on power
- Adaptive switches (Moe & Hedman, 1997)

Control with Low-Freq PWM

- Very easy to modulate power
- Tunable saturation limits

PWM Control with Tunable Saturation



Results

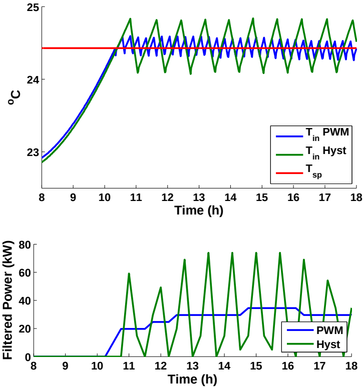
Verification with House Simulation Software

- 5-State house
- Inputs:
  - Outside Temperature
  - Solar radiation
  - Infiltration
  - Internal inputs

Comparison

- PWM system error – varies with sample period T
- Input power – linearizing quality of PWM

PWM Control of Full Simulation



Conclusion

Low Frequency PWM Benefits

- Simplified control of multi-stage compressors.
- Simplified load management.
  - Linearizing property improves system ID.
  - Energy consumption known prior to use.
  - Tunable saturation improves power limiting.
- Enables intelligent residential load management.

Contacts

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