# Residential Electricity Auction with Uniform Pricing and Cost Constraints

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# Introduction

Our Economic System Allocates Goods based on Cost

- Cost constraints are ubiquitous
- Only have a certain amount of money
- Want as much as you can get

Anecdotal Example: gasoline

- Bill buys \$20 of gas regardless
- Valentine's Day Week: $\$3.00/gal \rightarrow 6.67gal$
  $\Rightarrow$ Bill drives everywhere
- Memorial Day Weekend: $\$4.00/gal \rightarrow 5gal$
  $\Rightarrow$ Bill takes the bus

# Motivation

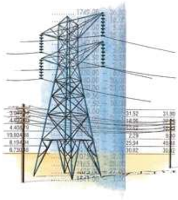
Real-Time Electricity Pricing

- Couple consumer price to markets
- Consumer response to price improves efficiency
- Argued for in (Borenstein, 2002)

State of RTP

- In CAISO territory
  - Time Of Use (TOU) programs in place
  - Prices fixed well ahead of time (i.e. years ahead)
  - Likelihood of RTP very high
- PJM already has a program

Enable *Automatic* Price Response



# Problem Description

Product Description

- Removed complexity of production cost
- Product is scarce (not enough for everyone)
- Product is infinitely divisible

Bidding Population

- Many bidders desiring item
- Bidder desires certain quantity
- Bidder will take less if cost too high

Research Goal: Find Auction Mechanism

- Allocate based on uniform price
- Mechanism should be fast
- Mechanism should be truthful

# Previous Work

Game Theoretic Approach (Nisan, Roughgarden, & Vazirani, 2007)

- Analyze how rational agents interact to achieve goals
- Algorithmic mechanism design concerned with developing the framework for interaction

Divisible goods auctions – Treasury Bonds

- Discriminatory pricing not better than uniform pricing, and it could be worse, (Wilson, 1979)
- "Collusion like equilibria" in divisible goods auctions under uniform pricing, (Back & Zender, 1993)
- "Collusion like equilibria" fixed with option of strategic supply withdrawal, (Back & Zender, 2001)
- Unwanted equilibria exist only under a continuous bidding strategy, (Kremer & Nyborg, 2004)

# Previous Work

Budget Constraints – Hard Constraints

- Single good auctions lose efficiency under budget constraints, (Maskin, 2000)
- No deterministic truthful mechanism to allocate multiple identical items to a multitude of budget constrained (hard) bidders, (Borgs, Chayes, Immorlica, Mahdian, & Saberi, 2005).
- Impossibility proof for multi-unit constrained budget (hard) auctions, (Dobzinski, Lavi, & Nisan, 2008)

Electricity auctions – Generation Scheduling

- VCG mechanism for electricity markets, (Hobbs, Rothkopf, Hyde, & ONeill, 2000)
- Uniform pricing not clearly better than discriminatory pricing, (Zhang, Jiao, Chen, & Ni, 2003)

# Clearing Algorithm – Preliminaries

Formalized Problem Description

- Divisible item $E^*$
- Many bidders ($n > 1$)
- Bidder ($i = 1, 2, ..., n$) has a private evaluation
  - Maximum desired quantity of the item, $\alpha_i$
  - Maximum unit price for that quantity, $\rho_i$
- Demand outstrips supply, $\sum_{i=1}^{n} \alpha_i > E^*$
- Bidder exhibits a *soft budget constraint*

Goal: Allocate Full Quantity to the Bidders, $A = a_1, a_2, ..., a_n$, at a Uniform Price $P$.

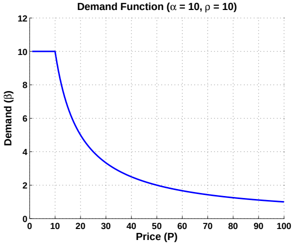
# Clearing Algorithm – Soft Budget Constraint

The bidder evaluations follow a *soft budget constraint*

## Definition

**Soft Budget Constraint** – A budget in which the bidder desires a given quantity $\alpha_i$ of an item, for a maximum price $\rho_i$, but will accept a proportionally lower quantity at a higher price. The actual quantity desired is represented by $a_i$, and the actual price is given by $P$. A *soft budget constraint* function, $\beta_i : P \longrightarrow a_i$, is succinctly represented by (1).

$$\beta(P) = \begin{cases} \frac{\rho_i \alpha_i}{P} & ; P \geq \rho_i \\ \alpha_i & ; P < \rho_i \end{cases} \tag{1}$$



# Clearing Algorithm – Proposed Mechanism

## Soft Budget Constrained Mechanism

$f : \{E^*, b_1, b_2, ..., b_n\} \rightarrow \{P, a_1, a_2, ..., a_n\}$

1. Order the bids ($b_i = \{\alpha_i, \rho_i\}$) based on the maximum price ($\rho_i$) in ascending order placing a fictitious bid, $b_{n+1} = \{0, \infty\}$, at the end, $\rho_1 \leq \rho_2 \leq ... \leq \rho_n \leq \rho_{n+1}$
2. Iterate on (2), starting from $k = 1$ until $\rho_{k-1} < P \leq \rho_k$.

$$\begin{aligned} k &\leftarrow k+1 \\ P &\leftarrow \frac{\sum_{i=1}^{k-1} \rho_i \alpha_i}{E^* - \sum_{i=k}^{n} \alpha_i} \end{aligned} \tag{2}$$

3. Compute the allocations ($a_i$) using Equations 3 and 4.

$$a_i = \frac{\rho_i \alpha_i}{P} \quad \text{for } i = 1, 2, ..., k-1 \tag{3}$$

$$a_i = \alpha_i \quad \text{for } i = k, k+1, ..., n \tag{4}$$

# Soft Budget Constrained Mechanism

How It Works – Examine computation for the total allocation, (5)

$$E^* = \sum_{i=k}^{n} \alpha_i + \frac{1}{P} \sum_{i=1}^{k-1} \alpha_i \rho_i \tag{5}$$

- Bidders $i = 1, 2, ..., k-1$ have their allocation reduced by their budget constraint
- Bidders $i = k, k+1, ..., n$ obtain their full desired allocation
- Good allocation occurs when the price $P$ only constrains the $i = 1, 2, ..., k-1$ bidders

Benefits

- Fast – computed in polynomial time
- Communication Efficient – single bid per bidder.

# Policy Consistency

Mechanism is Policy Consistent

- Bidders directly reveal true valuations
- Can't get better outcome by lying
- AKA Incentive Compatible or Truthful

### Definition

**Policy Consistency** – A mechanism $(f, b_1, b_2, ..., b_n)$ is called incentive compatible if for every player $i$, every $v_1 \in V_1$, $v_2 \in V_2$,..., $v_n \in V_n$ and every $\tilde{v}_i \in V_i$, if we denote $A = f(v_i, v_{-i})$ and $\tilde{A} = f(\tilde{v}_i, v_{-i})$, then $v_i(A) - p_i(v_i, v_{-i}) \geq \tilde{v}_i(\tilde{A}) - p_i(\tilde{v}_i, v_{-i})$ (Nisan, 2007).

Note: We will consider the case when the bid consists of truthful revelation with a hat and any other bid with a tilde.

# Policy Consistency of Mechanism

**Theorem**

*The Soft Budget Constraint Mechanism is policy-consistent when the bidders have soft budget constraints.*

**Sketch of Proof.**

- Define the nonlinear valuation function such that utility is zero along the budget constraint
- Consider constrained and unconstrained cases:
  - Case 1: Unconstrained ($i \geq k$)
    - Maximum allocation at a price below their maximum price.
    - $\tilde{u}_i \leq \hat{u}_i \implies$ no better off by lying!
  - Case 2: Constrained ($i < k$)
    - Price is larger than the maximum bid price, and the allocation is constrained.
    - $\tilde{u}_i \leq \hat{u}_i \implies$ no better off by lying!

$\square$

# Application

Residential Real-Time *Electricity* Markets

- Treat consumption over fixed time period
- Consider energy a scarce resource
- Auction to consumers in real time

Proposed Auction Format:

1. *Bid Call:* At predefined time before, the units submit bids, bids = {expected consumption, price willing to pay}.
2. *Clearing:* Compute clearing price
3. *Auction Period:* Units charged the clearing price for their consumption

Example System – PCT Market (Burke & Auslander, 2009)

- Clearing price computed every 15mins to meet aggregate consumption desires.
- Uses low frequency PWM (15min period) for actuation
- Submit bids based on expected consumption 5min before

# Discussion

Collusion Like Equilibria with Uniform Pricing

- Observed with arbitrary and continuous bidding
  - Arbitrary bidding allows for steep demand near equilibrium
  - Soft Cost Constraint *not* arbitrary
  - Soft Cost Constraint not steep around equilibrium.
- Produces low revenue equilibrium
  - Not directly considered for this treatment
  - Value determined by bidders' willingness to pay
  - Low revenue unlikely unless bidders undervalued the item, but then it would not be scarce

Impossibility Proof of Multi-Unit Auctions with Budgets

- Says that optimality and truthfulness are impossible
- Considered hard cost constraints
- Soft Budget Constraints accept some quantity even at a high price

# Conclusion

Research Overview

- Scarce resource auctions with divisible goods
- Soft budget constraints

Research Findings

- Policy consistent clearing algorithm
- Mechanism is fast
- Mechanism is communication efficient

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# Thank You

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